

Prospect of architectonic grammar reconstruction from dense 3D point clouds: Historical building information modeling (HBIM) of Guangdong cultural heritages

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Abstract:

Building information modeling (BIM) of cultural heritages, i.e., historic building information modeling (HBIM), advances the monitoring, maintenance, restoration, and virtual exhibitions of historical buildings. However, due to the elaborate styles and the unavoidable erosion and renovation, the reconstruction of HBIM from the prevalent raw data, such as point clouds and images, is very challenging, especially parametrical and semantic modeling. Recent studies have noticed the potential of architectonic grammar for facilitating parametric and semantic reconstruction. In this paper, we investigate the manual modeling of cultural heritage with the architectonic grammar and propose a roadmap consisting of four levels of automation, i.e., ‘calibration,’ ‘selection,’ ‘combination,’ and ‘generation,’ of the architectonic grammar reconstruction. Further quality improvement and cost analysis of these four levels show that ‘calibration’ and ‘selection’ are the most suitable options currently for real-world applications. This study inspires the future application of architectonic grammar to facilitate the parametric and semantic HBIM reconstruction and explores the prospect of a new HBIM reconstruction schema.

Keywords:

HBIM, BIM automation, cultural heritage, architectonic grammar, parametric modeling, building semantics, automated model reconstruction

1 Introduction to architectonic grammar reconstruction

Building information modeling (BIM) is a digital representation of physical and functional characteristics of a facility to enhance data interoperability and serves as the fundamental information infrastructure to facilitate data sharing, construction control, facility management, and decision making (NIBS 2015). When applied in cultural heritage, i.e., historic buildings, BIM is known as historic BIM (HBIM) (Murphy et al. 2009). HBIM has received much attention in the past decade due to its promising applications in heritage monitoring, maintenance, restoration, and virtual exhibitions (López et al. 2018).

However, HBIM is not as prevalent as its counterpart of modern buildings, and its automation supported by the software is still limited (López et al. 2018). The reconstruction of HBIMs could be much more challenging than that of BIMs. Because unlike modern buildings which are prone to be highly regular, concise, or compact, historic buildings are always highly elaborate, both in structures, e.g., the wooden composition of Shanxi Hanging

14 Temple, and decorations, e.g., the facades of Cathédrale Notre-Dame de Paris (Pocobelli et
15 al. 2018). Therefore, the complexity of geometry, semantics, and topology of heritages could
16 be very high. Furthermore, erosion and refurbishment are unavoidable in cultural heritages
17 during their long history, which improves the complexity of HBIM reconstruction.

18 Existing approaches to HBIM take advantage of 3D scanning and photography to
19 capture cultural heritages' surface information (Murphy et al. 2009; Quattrini et al. 2015).
20 Once captured, mesh models can be created by triangulation from point clouds or performing
21 structure-from-motion (SfM) algorithms on images, which is mature and automatic. Next, to
22 create semantic and parametric HBIMs, interactive and automatic solutions have been
23 investigated. For example, Murphy et al. (2009) introduced an HBIM system to map the BIM
24 objects from a parametric object library onto point clouds. However, the conversion from
25 point clouds or mesh models to parametric components of these HBIM systems still requires
26 much manual effort. Moreover, BIM semantics recognition by segmentation is a typical
27 schema of automatic reconstruction, which could be further categorized into (i) heuristic and
28 (ii) learning approaches (Bassier et al. 2019; Chen et al. 2019). However, segmentation with
29 heuristic rules is limited to simple geometry shapes (Musialski et al. 2013). Moreover,
30 segmentation by learning relies on burdensome manual annotations for training. Furthermore,
31 both paradigms of segmentation are sensitive to data imperfections, e.g., occlusion and clutter
32 in point clouds or mesh models.

33 Recently, architectonic grammar is exploited for the parametric reconstruction of
34 buildings as a segmentation-free approach. The architectonic grammar regularizes the
35 expressions of elements, forms, and styles from the ground plan to the rooftop (Cole 2002)
36 and is presented in our built environments from the main structure to the smallest details. It
37 has some essential properties that facilitate parametric and semantic reconstruction of HBIM.
38 First, architectonic grammar is highly extensible to different architectural styles based on the
39 concept of meta-grammar. Traditional grammars could be found out in the cultural heritages;
40 meanwhile, some architectural modernists, such as *Frank Gehry* and *Zaha Hadid*, have
41 deviated their distinguishable ones. Secondly, architectonic grammar holds a hierarchy from
42 the main structures to the smallest details. More specifically, the grammar covers the
43 definitions of (i) parameters, (ii) geometric primitives, (iii) components, and (4) component
44 relations. Consequently, researchers and engineers can configure the most suitable hierarchy
45 level for HBIM reconstruction. For example, the HBIM system can automatically adjust the
46 parameters or select appropriate geometric primitives and parameters of the semantic

47 components. Thirdly, the architectonic grammar is compatible with some statistical reasoning
48 frameworks (Kalogerakis et al. 2012), advancing the automatic semantics recognition while
49 preserving the interpretation compared with some “black-box” segmentation-based
50 reconstruction.

51 This paper aims at proposing a roadmap for the future automation of the architectonic
52 grammar reconstruction from point clouds. First, we select a set of Guangdong cultural
53 heritage sites and collected dense and colorful 3D point clouds. A manual process then
54 reflects how the architectonic grammars of the target heritage buildings can be organized into
55 the Grasshopper diagrams. A roadmap consisting of four levels of automation is presented in
56 contrast with the manual modeling results. We recommend the ‘calibration’ and the
57 ‘selection’ levels for practitioners.

58 **2 The case of Guangdong cultural heritage**

59 A pilot study was conducted on a case in Sanxiang Town, Zhongshan City, Guangdong
60 Province, China, as shown in Figure 1. We focused on three cultural heritage sites’ colorful
61 point clouds scanned by a drone. The LiDAR point cloud includes one Tower named Wenge
62 and three watchtowers (486 MB compressed on disk), which were relatively complete and
63 uniformly sampled in the LASzip Compressed Lidar (.laz) format. The data set’s coordinate
64 system was WGS 84/UTM zone 49N (EPSG: 32649).



65
66 **Figure 1. Three study sites as circled around the Old Street, Sanxiang Town, Guangdong**

67 Wenge Pagoda, located in the Xuzai community of Sanxiang Town, was built in 1747 in
68 Qing Dynasty, which has a 273-year-long history as for now. The tower has five stories and
69 is 30 meters high, covering an area of 39 square meters, which was rebuilt three times in
70 1819, 1895, as well as 1984 and announced as a cultural relic protection unit in Zhongshan
71 City in 1990. A dense cloud of 29,606,820 colorful points were collected from the
72 photogrammetric model of drone photographs. The mean volume density of it is 7272552.5
73 pts/m³.



74
75 **Figure 2. Dense point clouds of cultural heritage buildings, with a density of >10,000 pts/m². (a) Wenge Pagoda**
76 **(29,606,820 points), (b) a watchtower (13,571,861 points), (c) twin watchtowers (20,853,279 points)**

77 Watchtowers as a typical historic site in Sanxiang Town are scattered in the old streets
78 and alleys, especially in Baishi Village. They were built from the end of the Qing Dynasty to
79 the Republic of China for military use. These watchtowers have been listed in the category of
80 Historical Buildings in Zhongshan City in 2009 and listed in the Historical and Cultural
81 Protection Areas of Sanxiang Town. Three watchtowers of them were scanned and collected
82 in the point cloud datasets. The first set with one tower has 13,571,861 colorful points while
83 the second set including two watchtowers of 20,853,279 colorful points. The point densities
84 in the datasets are over 10,000 pts/m².

85 **3 Baseline parametric modeling**

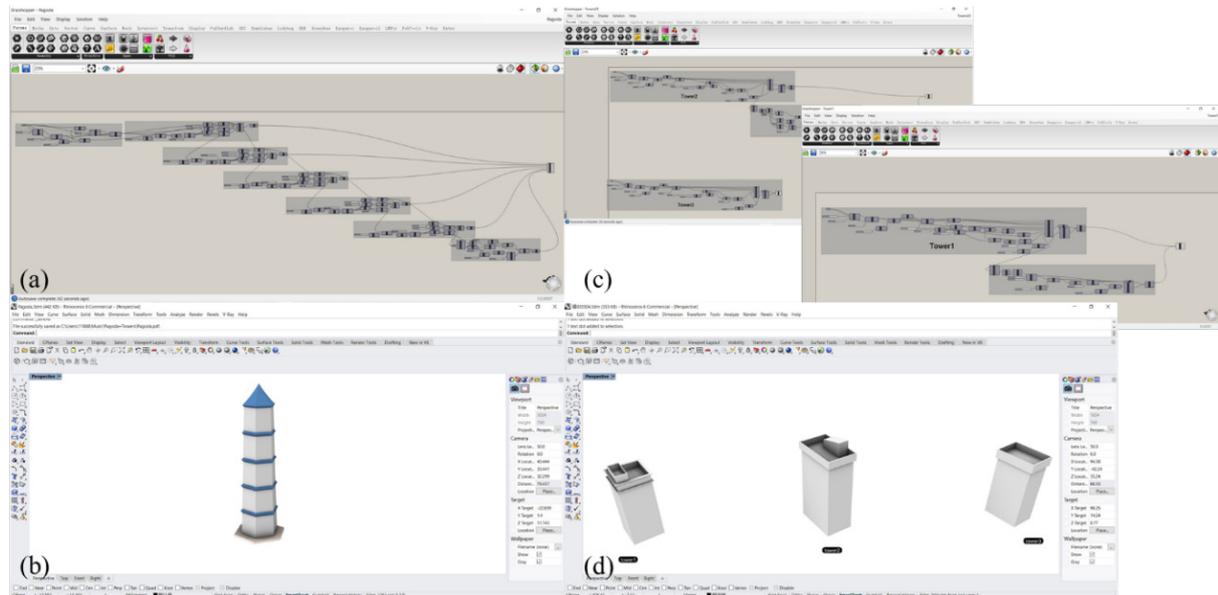
86 We utilized the Grasshopper, a built-in modeling language and plug-in on the Rhino platform
87 (ver. 6), for the manual parametric modeling. Grasshopper is a non-uniform rational basis

88 spline (NURBS) based on visual programming language and 3D modeling software. The
89 value of Grasshopper lies in parametric modeling and human-machine interactive design.
90 Besides, Rhino can create, edit, analyze, and transform NURBS curves, surfaces, and entities
91 in the aspect of complexity, angle, or size—though Rhino 6 needs relatively high computer
92 configuration. The laptop for the experiment runs a Windows 10 (64 bits) on Intel i9-9980HK
93 CPU, 16GB memory, and NVIDIA Quadro T2000 GPU.

94 First, the dense point clouds were converted from the data set to ASTEM E57 (.e57)
95 format and Wavefront object (.obj) for Rhino's use. The three sets of point clouds were
96 centered on the origins. Grasshopper was utilized to rebuild the reference points from the
97 point cloud data. In sequence, it is used to be the center point of the reference plane in the
98 model. Then, the shape's contour line or curve battery by setting the plane figure's relative
99 length details and shows different its location through translation or rotation. Then, the
100 contour line or curve battery of the shape may move up or down, expand or shrink through
101 number battery to form other flat figures of different heights or sizes needed. After that, the
102 shape surfaces can form from external contour lines or curves through covering surfaces. The
103 operator might also blast curves and then extract, and loft surfaces separately if some surfaces
104 are not needed ultimately. Eventually, a series of planes from every module is made up of
105 generated planes in the module and then are composed of the whole model surface.

106 The architectonic grammar diagram of the pagoda in Grasshopper, as shown in Figures
107 3a and 3b, consists of six modules. The base and the first layer of the pagoda presents in the
108 first module. Firstly, the reference anchor point (0,0,0) needs to be found from the 3D could
109 point model. Secondly, the base's hexagonal contour line of with 5 meters on each side is
110 generated with the reference point as the center point of the graph, then it rotates 15 degrees
111 counterclockwise and moves up 0.4 meters. Then, the hexagonal contour line of the base goes
112 to move up 0.4 meters and do the same generation and rotation again. In sequence, the first
113 layers' hexagonal contour line is produced by moving up or down 0.35 meters while
114 expanding to 1.1 times or shrinking 0.95 times. Lastly, the above generated hexagonal
115 contour lines are combined into a line composition and then the set of lines lofting into the
116 first layer plane together with the base one. From the first layer to the second one, the top
117 hexagonal contour line of the first layer is lifted 4.5 meters to become the middle one of the
118 first layer. Next, the same steps are continued. Then, the generation from the first to the
119 second is applicable to the third, fourth, and fifth layers. In the tower top module, the spire
120 performs by lifting the fifth layer's reference anchor point by 3.6 meters. The six points of the

121 middle hexagonal contour line of the fifth layer move up 0.6 meters and then shrinks to 0.8
122 times to be the reference points for the tower roof design. In the end, the spire's point is
123 duplicated to generate the circular reference line, and then the six planes are set out to get the
124 top roof of the tower.



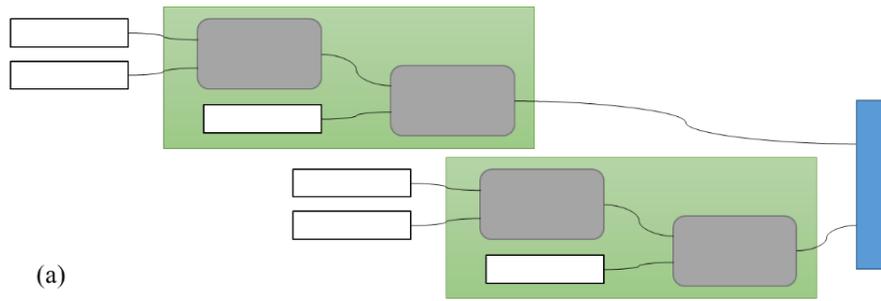
125
126 **Figure 3. Grasshopper diagram and parametric models. (a) Diagram for Wenge Pagoda, (b) 3D view of (a), (c)**
127 **diagrams for the watchtowers, (d) 3D view of (c)**

128 The architectonic grammar diagram of the first tower in Grasshopper, as shown in
129 Figure 3c and 3d, consists of 2 modules. The body layer of the tower presents in the first
130 module. Firstly, the reference anchor point (0, 0, 0) needs to be found from the 3D could
131 point model. Next, the base's rectangular contour line is generated with the reference point as
132 the center point of the graph, and then it rotates 35 degrees counterclockwise and moves up 8
133 meters to generate the rectangular contour line of the lower top layer. Next, the top layer's
134 rectangular contour lines are produced by moving up 0.1 meters, up 0.1 meters, or down
135 0.895 meters while expanding to 1.1 times or shrinking 0.95 times. Lastly, the rectangles
136 cover, and lines are lofting into planes. Therefore, the surface composition of the tower's
137 body layer performs in the model. In the second module, the floor of the roof covers through
138 the principle called the three points to form a plane. Next, the floor of the affiliated small
139 house generates by the above reference point and setting the length of X and Y. Finally, a set
140 of horizontal rectangular contour lines of the house are produced through moving up 1.8
141 meters or 0.895 meters meanwhile shrinking 0.9 times. Eventually, a series of rectangles
142 cover and lines lofting into planes. Similarly, in the last Grasshopper diagram, the second
143 tower's modeling process is divided into two modules, and the third towers are divided by 1
144 module. The sequence of the first tower's modeling is also adaptive to these two.

145 These restoration models of ancient buildings in Guangdong through computational
146 design software preliminarily perform the buildings' main body shape. Therefore, what is
147 fundamental and essential in reserving while reconstructing the cultural heritage field is that
148 the broken and incomplete physical architectural model shows again in public in the form of
149 virtual architectural models through novel digital tools. Furthermore, one of the usual
150 computational design methods is parametric design, a design process in which the
151 engineering itself is programmed as a function and a process. The design process is
152 automatically realized by modifying the initial conditions and obtaining the engineering
153 results by computer calculation.

154 **4 A roadmap to the automation**

155 The automation of architectonic grammar reconstruction can be projected and classified into
156 four levels, as shown in Figure 4. We noticed that the diagrams in Grasshopper were
157 comprised of four types of grammar components, i.e., (i) parameters, (ii) geometric
158 primitives, (iii) components, and (iv) relations to new components. Therefore, the most
159 straightforward automation is to let the machine fine-tune the parameters, while the whole
160 diagram structure designed manually remains unchanged. The most challenging automation
161 level is automatic incremental design or revision of new components, while no apparent work
162 demand is there for human modelers. Note that the automation roadmap and the levels are
163 independent of the Grasshopper + Rhino and compatible with other parametric design tools
164 such as Dynamo + Revit.



(a)

Level of diagram automation for BIM reconstruction	Parts of the grammar (: by human, 🤖 : by computer)			
	Parameters	Components	Primitives	New comp.
Level 1: Calibration	🤖	👤	👤	👤
Level 2: Selection	🤖	🤖	👤	👤
Level 3: Combination	🤖	🤖	🤖	👤
Level 4: Generation	🤖	🤖	🤖	🤖

(b)

Figure 4. Four levels of prospect of automation for architectonic grammar reconstruction. (a) A general diagram, (b) table of diagram automation

The first automation level is ‘calibration.’ At this level, the whole grammar structure still comes from an experienced modeler’s manual work. The structure aims to reflect what components and primitives are there in the rough parameters of locations and sizes. The machine will do the parameters calibration, automatically. In this way, the human resource can be partially relieved from the laborious effort on fine-tuning the small digits in the parameters. Similar parameter optimization approaches are known well in the BIM performance fine-tuning (Asl et al. 2015) as well as HBIM (Bienvenido-Huertas et al. 2019). The saving will be more considerable if the parameters are interconnected—so that one small change in a parameter leads to impacts to another parameter.

The next level is ‘selection.’ A selection-level grammar reconstruction inherits the parameters automation part of the first level. Additionally, the components are selected automatically from an available library. For the manual work, the modelers first need to prepare a big enough component library—like the BIM component and resources libraries. Then, a sketch diagram of known relations of major unknown components can guide the machine to search for the best-fit instances in the library. On every trial, the first calibration level will be called to tell the best fitness. Overall, the machine runs in a trial-and-error fashion. For instance, Xue et al. (2019a) show that automatic ‘semantic registration’ of 8 furniture BIM components to a noisy point cloud. The semantic registration first performed such a ‘selection’ automation, then calibrated the three parameters, i.e., x, y, and heading

187 direction. According to the experiments in Xue et al. (2019a), over 98% of modeling time
188 was saved.

189 The ‘combination’ level elevates the selection level by evolving the components. Every
190 as-designed component in the library consists of a system of geometric primitives. A
191 combination-level automation machine evolves these primitives to the best-fit primitives
192 through iterated evolutionary computation. For example, suppose the six sides of Wenge
193 Pagoda in Figure 2a are slightly different (e.g, deformations within 5 mm), while the selected
194 components by a Level-2 machine are perfectly symmetric. In that case, the Level-3
195 combination operation will try to select the related geometric primitives inside of the best-fit
196 components for better fitting to the measurement. The combination-level reconstruction
197 improves the accuracy of the Level-2 selection results.

198 The fully automatic level is classified as ‘Generation’ in this framework. The *a*
199 *priori* setting of relations among major components is automated at this level. As a result, the
200 relation modeling and arbitrarily new components are created by the machine rather than
201 human export. However, a combinatorial explosion of the computational load growth is
202 expected, due to the complicated and nested variables in the four levels. In the near future,
203 the authors are not optimistic about seeing massive applications of this level to modeling
204 cultural heritages.

205 The selection of an appropriate level can be based on the trade-off between marginal
206 quality and cost. It means if both quality and cost are improved by an automated method, it is
207 strongly recommended. Furthermore, the most recommended level is equipped with
208 maximum bi-objective gains. Because usually, the quality may increase along with the
209 automation level, while the cost is in the opposite direction. For example, Table 1 shows an
210 assumed trade-off table for a company. When we use the “MIN()” function to measure the
211 bottom-line gain, the ‘selection’ is the best level. The highest level may not be the best level.
212 The authors wish to see progressive research and development in the next couple of decades,
213 regarding the drivers and barriers. There is no need to target the highest level at the very
214 beginning blindly.

215

216 **Table 1. Example trade-off table and recommendation for a company**

Level	Name	Quality gain (e.g., accuracy, innovation, semantics, etc.)	Cost gain (e.g., money, time, effort)	Recommendation (e.g., based on MIN(quality, cost))
1	Calibration	★	★★★	
2	Selection	★★	★★	✓
3	Combination	★★★	(Cost increased)	
4	Generation	★★★	(Cost increased)	

217 **5 Conclusion**

218 The automation of parametric and semantic HBIM reconstruction remains a very challenging
219 topic to date. New schemas are desirable to improve this automation without increasing the
220 cost. Architectonic grammar shows excellent potentials for HBIM reconstruction in recent
221 studies. Therefore, we investigated a pilot case and a roadmap to inspire the future
222 automation of architectonic grammar reconstruction from point clouds. The manual modeling
223 of the selected Guangdong cultural heritages demonstrates how the architectonic grammars of
224 the historic buildings can be organized as Grasshopper diagrams. Next, a roadmap described
225 the four levels of automation is proposed. The quality improvement and cost of these four
226 levels are also analyzed. Consequently, ‘calibration’ and ‘selection’ levels are recommended
227 for practitioners based on the prospect of future research.

228 Following our roadmap, we will further investigate and develop the automatic
229 ‘calibration’ and ‘selection’ methods based on architectonic grammar, and search for
230 opportunities to attack the ‘combination’ and ‘generation’ levels of architectonic grammar
231 reconstruction. Along with the architectonic grammar reconstruction, the geometric,
232 semantic, and topological definitions in BIM and HBIM will be exploited and formulated.
233 Moreover, advanced evolutionary computation algorithms and design knowledge suitable for
234 solving such non-linear and expensive optimization problems, such as derivative-free
235 optimization (DFO) and Gestalt principles (Xue et al. 2019b; 2020), will be employed in our
236 automatic reconstruction.

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243 Research design, proofreading and revision.

244 **References**

- 245 Asl, M. R., Zarrinmehr, S., Bergin, M. & Yan, W. (2015). BPOpt: A framework for BIM-
246 based performance optimization. *Energy and Buildings*, 108, 401-412.
247 doi:10.1016/j.enbuild.2015.09.011
- 248 Bassier, M., Van Genechten, B. & Vergauwen, M. (2019). Classification of sensor
249 independent point cloud data of building objects using random forests. *Journal of*
250 *Building Engineering*, 21, 468-477. doi:10.1016/j.jobbe.2018.04.027
- 251 Bienvenido-Huertas, D., Nieto-Julián, J. E., Moyano, J. J., Macías-Bernal, J. M. & Castro, J.
252 (2019). Implementing artificial intelligence in H-BIM using the J48 algorithm to manage
253 historic buildings. *International Journal of Architectural Heritage*, 14(8), 1148-1160.
254 doi:10.1080/15583058.2019.1589602
- 255 Chen, J., Kira, Z. & Cho, Y. K. (2019). Deep learning approach to point cloud scene
256 understanding for automated scan to 3D reconstruction. *Journal of Computing in Civil*
257 *Engineering*, 33(4), 04019027. doi:10.1061/(ASCE)CP.1943-5487.0000842
- 258 Cole, E. (2002). *The grammar of architecture*. Boston, USA: Bulfinch Press.
- 259 Kalogerakis, E., Chaudhuri, S., Koller, D. & Koltun, V. (2012). A probabilistic model for
260 component-based shape synthesis. *ACM Transactions on Graphics*, 31(4), 55.
261 doi:10.1145/2185520.2185551
- 262 López, F. J., Leronés, P. M., Llamas, J., Gómez-García-Bermejo, J. & Zalama, E. (2018). A
263 Review of Heritage Building Information Modeling (H-BIM). *Multimodal Technologies*
264 *and Interaction*, 2(2), 21. doi:10.3390/mti2020021
- 265 Murphy, M., McGovern, E. & Pavia, S. (2009). Historic building information modelling
266 (HBIM). *Structural Survey*, 27(4), 311-327. doi:10.1108/02630800910985108
- 267 Musialski, P., Wonka, P., Aliaga, D., Wimmer, M., van Gool, L. & Purgathofer, W. (2013).
268 A Survey of Urban Reconstruction. *Computer graphics forum*, 32(6), 146-177.
269 doi:10.1111/cgf.12077
- 270 NIBS. (2015). *Standard National BIM Standard - United States Version 3*. National Institute
271 of Building Sciences. Retrieved September 24, 2019, from <https://bit.ly/33gTWZ4>

- 272 Pocobelli, D. P., Boehm, J., Bryan, P., Still, J. & Grau-Bové, J. (2018). BIM for heritage
273 science: a review. *Heritage Science*, 6(1), 30. doi:10.1186/s40494-018-0191-4
- 274 Quattrini, R., Malinverni, E. S., Clini, P., Nespeca, R. & Orlietti, E. (2015). From TLS to
275 HBIM. High quality Semantically-aware 3D modeling of complex architecture. *The
276 International Archives of the Photogrammetry, Remote Sensing and Spatial Information
277 Sciences. XL-5/W4*, pp. 367-374. Avila, Spain: ISPRS. doi:10.5194/isprsarchives-XL-5-
278 W4-367-2015
- 279 Xue, F., Lu, W., Chen, K. & Zetkunic, A. (2019a). From Semantic Segmentation to Semantic
280 Registration: Derivative-Free Optimization-Based Approach for Automatic Generation
281 of Semantically Rich As-Built Building Information Models from 3D Point Clouds.
282 *Journal of Computing in Civil Engineering*, 33(4), 04019024.
283 doi:10.1061/(ASCE)CP.1943-5487.0000839
- 284 Xue, F., Lu, W., Chen, Z. & Webster, C. J. (2020). From LiDAR point cloud towards digital
285 twin city: Clustering city objects based on Gestalt principles. *ISPRS Journal of
286 Photogrammetry and Remote Sensing*, 167, 418-431.
287 doi:10.1016/j.isprsjprs.2020.07.020
- 288 Xue, F., Lu, W., Webster, C. J. & Chen, K. (2019b). A derivative-free optimization-based
289 approach for detecting architectural symmetries from 3D point clouds. *ISPRS Journal of
290 Photogrammetry and Remote Sensing*, 148, 32-40. doi:10.1016/j.isprsjprs.2018.12.005
291
292